Digital Actuarial Resources

Practice Test Questions for

SOA Exam FM / CAS Exam 2

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Practice Test Questions

Exam FM: Financial Mathematics
Society of Actuaries

Created By: Digital Actuarial Resources

(Sample Only – Purchase the Full Version)

Introduction:

This guide from Digital Actuarial Resources (DAR) contains sample test problems for Exam FM offered through the Society of Actuaries. The book has over 200 practice questions to test your knowledge of the principles of interest rates. The problems encompass applications of interest rates in annuities, bonds, loans, and stocks. The set of questions is very comprehensive and attempts to cover all major topics featured on the actual test. Nearly all of these questions are math-based. Some of the examples require calculus, while others entail advanced algebra.

You should expect to spend several days taking this test. There is no time limit. You can use your notes, textbooks, other actuaries, and whatever will help you answer the questions. These problems test your actuary skills and also attempt to teach you something. If you can correctly answer 75% of the questions (about 150 problems), you are prepared for the actual test.

The first half of this guide contains the practice test questions. You should use your own scratch paper when taking the test so that you can retake it several times. The detailed solutions start on page 41. If you find any errors in the solutions or would like to debate an answer, please contact the Digital Actuaries.

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Exam FM, Practice Test Questions
(1.) The balance in a savings account at time 4 is $12,000. At time 3, the balance was $11,600. What is the effective rate of interest during the fourth period?

(16.) If the nominal rate of interest compounded monthly is 7.2%, what is the annual effective rate of interest?

(19.) An insurance company expects to collect $450 in premiums from a client in 7 months. The rate of interest compounded semi-annually is 5%. What is the present value of the premiums?

(23.) Suppose the force of interest is defined by the following equations:

\[
\delta_r = \begin{cases} 
0.014r, & \text{for } 0 \leq r \leq 7 \\
0.001r^2 - 0.005r, & \text{for } 7 < r \leq 10 
\end{cases}
\]

What is the present value of $1 to be paid in 10 years?

(33.) What is the accumulated value in 10 years of an annuity paying $200 at the end of each year, with an annual effective interest rate of 4.3%?

(47.) What is the price of a perpetuity immediate, ignoring mortality, that pays $60,000 every 5 years with interest convertible annually at 5.4%?
(68.) An inflation-indexed retirement pension begins with a payment of $30,000 at the end of the first year. Each year, the payout rises by 4%. The employee is currently age 40, and payments begin at 65 and lasts exactly 20 years (guaranteed). If the company wishes to fully fund the annuity today, what will it cost? Assume $i$ is 7% per annum.

(72.) Suppose a project has the following inflows and outflows:

**Inflows:**

<table>
<thead>
<tr>
<th>$t$</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4,000</td>
</tr>
<tr>
<td>5</td>
<td>4,500</td>
</tr>
<tr>
<td>6</td>
<td>8,000</td>
</tr>
<tr>
<td>10</td>
<td>11,000</td>
</tr>
</tbody>
</table>

**Outflows:**

<table>
<thead>
<tr>
<th>$t$</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10,000</td>
</tr>
<tr>
<td>1</td>
<td>2,000</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
</tr>
</tbody>
</table>

What is the NPV of the project? Let $i = 6.5\%$.

(84.) Use the time-weighted method in the next example:

The beginning account balance at time 0 is $210,000. Deposits to the account follow this schedule:

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_t'$</td>
<td>4,000</td>
<td>7,000</td>
<td>7,500</td>
</tr>
</tbody>
</table>
Interest credits follow this schedule:

<table>
<thead>
<tr>
<th>t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>3,500</td>
<td>6,200</td>
<td>-1,500</td>
<td>-2,100</td>
</tr>
</tbody>
</table>

What is the annual effective yield rate?

(137.) A bond has a redemption value of 1. The face value is 1.018, the coupon rate is 9.8% with annual coupons, and the term is 30 years. If the principal adjustment at time 17 is 0.021, find the YTM.

(153.) Consider a bond with annual coupons. The book value after 5 years is $780. The original owner of the bond resells it 5 years and 329 days after its original sale. The flat price using the practical method is $843 at the time of the second sale. Find the flat price using the theoretical method at the new time. Use the actual/actual method for interest.

(162.) On November 1, 2200, an account opens with a balance of $50,000. On January 1, 2201, the owner adds $6,000. On March 1, 2201, the owner adds $X. On October 1, 2201, the owner withdraws $8,500. The final balance on November 1, 2201, is $54,350. Find X, if the annual effective interest rate over the year is 7.15% computed with the dollar-weighted method.
Solutions

(1.)

\[ i_4 = \frac{A(4) - A(3)}{A(3)} = \frac{12,000 - 11,600}{11,600} = \frac{400}{11,600} = 0.03448 \]

(16.)

\[ i^{(12)} = 0.072 \]

\[ 1 + i = \left(1 + \frac{i^{(12)}}{12}\right)^{12} \]

\[ 1 + i = \left(1 + \frac{0.072}{12}\right)^{12} \]

\[ 1 + i = 1.006^{12} \]

\[ 1 + i = 1.07442 \]

\[ i = 0.07442 = 7.442\% \]

(19.)

\[ i^{(2)} = 0.05 \]

\[ 1 + i = \left(1 + \frac{i^{(2)}}{2}\right)^2 \]

\[ 1 + i = 1.025^2 \]

\[ 1 + i = 1.050625 \quad \rightarrow \quad i = 0.050625 \]

\[ 1.050625 = \left(1 + \frac{i^{(12)}}{12}\right)^{12} \]

\[ 1.0041239 = 1 + \frac{i^{(12)}}{12} \]

\[ 0.0041239 = \frac{i^{(12)}}{12} \]

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\[ i^{(12)} = 0.049487 \]

\[
p.v. = 450 \left( 1 + \frac{0.049487}{12} \right)^{-12(7/12)}
\]

\[
= 450 \times 1.0041239^{-7}
\]

\[
= \$437.22
\]

(23.)

\[
p.v. = a^{-1}(10) = e^{-\left[ \int_0^7 0.014r \, dr \right] - \left[ \int_7^{10} 0.001r^2 - 0.005r \, dr \right]}
\]

\[
\int_0^7 0.014r \, dr = \left[ 0.007r^2 \right]_0^7 = 0.007 \times 7^2 = 0.343
\]

\[
\int_7^{10} 0.001r^2 - 0.005r \, dr = \left[ 0.000333r^3 - \frac{0.005}{2}r^2 \right]_7^{10}
\]

\[
= (0.000333 \times 10^3 - 0.0025 \times 10^2) - (0.000333 \times 7^3 - 0.0025 \times 7^2)
\]

\[
= (0.333 - 0.25) - (0.11433 - 0.1225)
\]

\[
= 0.08333 + 0.00817 = 0.091503
\]

\[
p.v. = e^{-0.091503} = \$0.64759
\]

(33.)

\[
ac.v. = s_{100.045} = \frac{(1 + 0.043)^{10} - 1}{0.043} * 200 = \$2,434.89
\]

(47.)

\[
Price = \frac{1}{i \times s_{50.054}} * 60,000
\]

\[
s_{50.054} = \frac{1.054^5 - 1}{0.054} = 5.57
\]

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Calculating present value (p.v.)

\[ p.v. = \frac{1}{0.054 \times 5.57} \times 60,000 \]

\[ p.v. = \$199,481.35 \]

(68.)

\[ k = 0.04 \]

\[ p.v. = v^{20} \times 30,000 \times \frac{1 - \left(\frac{1.04}{1.07}\right)^{20}}{0.07 - 0.04} \]

\[ = 0.18425 \times 30,000 \times 14.45907 \]

\[ = \$79,922.52 \]

(72.)

Net Present Value (NPV) = (p.v. of inflows) – (p.v. of outflows)

\[ p.v. \text{ of inflows} = 4,000v^2 + 4,500v^5 + 8,000v^6 + 11,000v^{10} \]

\[ = 3,527 + 3,284 + 5,483 + 5,860 \]

\[ = \$18,154 \]

\[ p.v. \text{ of outflows} = +10,000 + 2,000v^4 + 3,000v^6 \]

\[ = +10,000 + 1,878 + 2,332 \]

\[ = \$14,210 \]

\[ NPV = 18,154 - 14,210 \]

\[ NPV = \$3,944 \]

(84.)

<table>
<thead>
<tr>
<th>Time:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution:</td>
<td>0</td>
<td>4,000</td>
<td>7,000</td>
<td>7,500</td>
<td></td>
</tr>
<tr>
<td>Fund Value:</td>
<td>210,000</td>
<td>213,500</td>
<td>223,700</td>
<td>229,200</td>
<td>234,600</td>
</tr>
<tr>
<td>Compound Rate:</td>
<td>1 + j_1</td>
<td>1 + j_2</td>
<td>1 + j_3</td>
<td>1 + j_4</td>
<td></td>
</tr>
</tbody>
</table>
\[1 + j_1 = \frac{B_1'}{B_0' + C_0'} = \frac{213,500}{210,000 + 0} = 1.01667\]

\[1 + j_2 = \frac{B_2'}{B_1' + C_1'} = \frac{223,700}{213,500 + 4,000} = 1.02851\]

\[1 + j_3 = \frac{B_3'}{B_2' + C_2'} = \frac{229,200}{223,700 + 7,000} = 0.9935\]

\[1 + j_4 = \frac{B_4'}{B_3' + C_3'} = \frac{234,600}{229,200 + 7,500} = 0.99113\]

\[(1 + i)^4 = 1.01667 \times 1.02851 \times 0.9935 \times 0.99113\]

\[(1 + i)^4 = 1.02964\]

\[i = 0.00733\]

\[(137.)\]

\[g = \frac{Fr}{C} = 0.09976\]

\[PA_{17} = 0.021 = (g - i) \times v^{30-17+1}\]

\[0.021 = (0.09976 - i) \times (1 + i)^{-14}\]

\[f(j) = (0.09976 - i)(1 + i)^{-14} - 0.021\]

Let \(j_1 = 0.05\), \(j_2 = 0.06\)

\[\Delta j = 0.01, \quad f(j_1) = 0.004132, \quad f(j_2) = -0.003414\]

\[j \approx 0.05 + 0.01 \left( \frac{-0.004132}{-0.003414 - 0.004132} \right)\]

\[j = 0.0555\]
(153.)

\[ B_5 = 780 \]

\[ k = \frac{329}{365} = 0.9014 \]

\[ B_{5.9014}^f \text{ with practical method} = 843 = B_5 \times (1 + 0.9014 \times i) \]
\[ 843 = 780(1 + 0.9014 \times i) \]
\[ i = 0.0896 \]

\[ B_{5.9014}^f \text{ with theoretical method} = 780 \times (1 + 0.0896)^{0.9014} = \$842.73 \]

(162.)

\[ i \approx \frac{I}{A + \sum C_i \times \text{duration}} \]

\[ A = \$50,000, \quad B = \$54,350, \quad C = X - 2,500 \]

\[ A + C + I = B \]
\[ 50,000 + X - 2,500 + I = 54,350 \]
\[ I = 6,850 - X \]

\[ 0.0715 = \frac{6,850 - X}{50,000 + 6,000 \times (5/6) + X \times (8/12) - 8,500 \times (1/12)} \]
\[ \downarrow \]
\[ 0.0715 = \frac{6,850 - X}{54,291.67 + (8/12) \times X} \]
\[ \downarrow \]
\[ 3,881.85 + 0.04767X = 6,850 - X \]
\[ 1.04767X = 2,968.15 \]

\[ X = \$2,833.10 \]